Stiffness Optimization of Orthotropic Laminated Composites Using Lamination Parameters

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The paper presents an efficient stiffness optimization approach on orthotropic laminated composites using lamination parameters. It is efficient to use lamination parameters as design variables since the stiffness components of laminated composites are expressed as a linear function of lamination parameters in the classical lamination theory. As an example of stiffness optimizations, the buckling optimization of orthotropic laminated cylindrical shells under combined loadings is treated using a mathematical programming method. The present approach shows good convergence behaviors for the optimum design and gives reliable optimization results.

Nomenclature

A_{ij}, D_{ij}	= in-plane and out-of-plane stiffness components, respectively
a_{ij}	= in-plane compliance components
F	= objective function
$f_{\min}$	= normalized buckling load
g	= constraint function
h	= thickness
h_i	= layer thickness fractions
k_x, k_y	= axial and lateral shell loading fractions, respectively
L, R	= length and radius of cylindrical shells, respectively
m	= number of axial half-waves
N, M	= stress and moment resultants, respectively
$\bar{N}_x$, $\bar{N}_y$	= axial and lateral shell loadings, respectively
n	= number of circumferential waves
P	= buckling load parameter
Q_{ij}	= reduced stiffness components
U, w	= stress function and deflection, respectively
U_i	= stiffness invariants
x, y, z	= axial, circumferential, and radial coordinates, respectively
α	= angle representing a feasible region of out-of-plane lamination parameters (Fig. 2)
ϵ, κ	= strains and changes of curvature, respectively
θ	= layer angle
λ	= normalized axial wave $(= m\pi R/L)$
ξ_1, ξ_2	= in-plane lamination parameters
ξ_9, ξ_{10}	= out-of-plane lamination parameters
$\bar{\sigma}$	= buckling stress
ϕ_{ij}, Φ_1, Φ_2	= defined in Eq. (13)

Presented in part as Paper 88-2332 at the AIAA/ASME/ASCE/AHS 29th Structures, Structural Dynamics, and Materials Conference, Williamsburg, VA, April 18–20, 1988; received Nov. 8, 1989; revision received June 14, 1990; accepted for publication June 14, 1990.

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Introduction

THE stiffness characteristics of laminated composites depend strongly on ply angles of the laminate. Therefore, it is important to tailor laminate configurations of laminated composites.

Introduction of lamination parameters is efficient and reliable in the stiffness optimization of laminated composites. It is known[1] that the stiffness characteristics of laminated composites based upon the classical lamination theory are governed by 12 lamination parameters and four independent stiffness invariants. In the orthotropic laminates, eliminating the coupling effects, the number of independent lamination parameters is reduced to four. The stiffness components of laminated composites are expressed as a linear function with respect to lamination parameters. Thus, it is an efficient approach to use lamination parameters as design variables in the stiffness optimization problems of orthotropic laminated composites.

The lamination parameters were first presented as geometric factors by Tsai et al.[1] One of the authors[2] of this paper has used the lamination parameters as design variables in the buckling optimization of orthotropic laminated plates where the buckling characteristics were governed by two out-of-plane lamination parameters. The design method for tailoring the mechanical properties of the laminated composites has been developed by Miki[3,4] and Fukunaga et al.[5,6] The feasible regions of the in-plane or out-of-plane lamination parameters have been examined and the determining method for the laminate configurations corresponding to the in-plane or out-of-plane lamination parameters has been obtained. The optimization problems governed by two in-plane or out-of-plane lamination parameters can be solved easily by using a mathematical programming method.

Some research works[7–9] have been made on maximizing the buckling load of laminated cylindrical shells by tailoring the laminated configurations. Nshanian et al.[7] and Hirano[8] have applied a mathematical programming algorithm to determine the optimal ply angle variation through the thickness, where they used the ply angles as design variables. This approach has the danger of encountering numerous minima in the design space, since the buckling load for laminated cylindrical shells is highly nonlinear with respect to the ply angles. Onoda[9] has obtained the optimal laminate configurations for the axial buckling of laminated cylindrical shells using 12 lamination parameters. He has shown that one of the optimal laminate configurations to maximize the compressive buckling load is an isotropic laminate with respect to both in-plane and out-of-plane stiffnesses. Although his contribution has been great for the axial buckling of laminated cylindrical shells, he has not discussed the fundamental properties of lamination parameters themselves, which are indispensable for determining the optimal lamination parameters of laminated cylindrical shells under combined loadings.

The present paper shows the relation between the four lamination parameters governing the stiffness characteristics of orthotropic laminates and also shows the determining method of laminate configurations corresponding to the lamination parameters. On the basis of these results, the buckling characteristics of orthotropic laminated cylindrical shells under combined loadings are discussed. The optimal laminate configurations to maximize the buckling load are obtained using the mathematical programming method. The derivatives of the buckling load with respect to design variables are evaluated by a central difference approximation, which gives a simple but effective evaluation of the gradients of the multiple eigenvalues. It is shown that several initial points in the optimization converge to almost the same optimum point for various kinds of loading cases.

Lamination Theory and Lamination Parameters

Classical Lamination Theory

We consider the generalized symmetric and balanced laminate of $[(\pm\theta_1)_{h_1}/(\pm\theta_2)_{h_2}/\cdots/(\pm\theta_n)_{h_n}]_s$, where $\pm\theta_i$ and h_i, respectively, denote the layer angle and thickness of the ith layer. From the assumption of the balanced laminate, the thickness of $+\theta_i$ layer is same as that of the $-\theta_i$ layer. For simplicity, the coupling terms of the bending-twisting, D_{16} and D_{26}, are neglected; then the symmetric and balanced laminate can be regarded as an orthotropic laminate.

In the classical lamination theory, the constitutive equation of orthotropic laminates is given by

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & 0 \\ 0 & D \end{bmatrix} \begin{Bmatrix} \epsilon \\ \kappa \end{Bmatrix} \tag{1}$$

where N and M, respectively, denote the stress and moment resultants; ϵ and κ, respectively, the strains and the curvature changes at the midplane; A_{ij} and D_{ij}, respectively, represent the in-plane stiffnesses and the out-of-plane stiffnesses.

Introducing the stiffness invariants and the lamination parameters, A_{ij} and D_{ij} can be expressed as follows[1]:

$$\begin{Bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \end{Bmatrix} = h \begin{bmatrix} 1 & \xi_1 & \xi_2 & 0 & 0 \\ 0 & 0 & -\xi_2 & 1 & 0 \\ 1 & -\xi_1 & \xi_2 & 0 & 0 \\ 0 & 0 & -\xi_2 & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \tag{2}$$

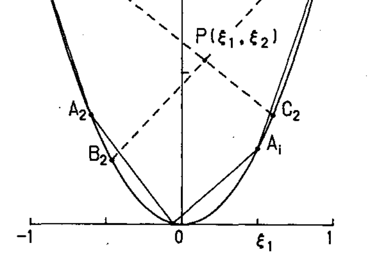


Fig 1 Characteristics of lamination parameters (ξ_1,ξ_2).

$$\begin{Bmatrix} D_{11} \\ D_{12} \\ D_{22} \\ D_{66} \end{Bmatrix} = \frac{h^3}{12} \begin{bmatrix} 1 & \xi_9 & \xi_{10} & 0 & 0 \\ 0 & 0 & -\xi_{10} & 1 & 0 \\ 1 & -\xi_9 & \xi_{10} & 0 & 0 \\ 0 & 0 & -\xi_{10} & 0 & 1 \end{bmatrix} \begin{Bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{Bmatrix} \tag{3}$$

where h is the thickness of the plate. The stiffness invariants U_i ($i = 1,2, \ldots, 5$) used here[10] are slightly different from Tsai's description.[1] The lamination parameters ξ_1, ξ_2, ξ_9, and ξ_{10} are written as follows:

$$\xi_1 \equiv \frac{1}{h}\int_{-h/2}^{h/2} \cos 2\theta(z)\, dz = \int_0^1 \cos 2\theta(u)\, du$$

$$\xi_2 \equiv \frac{1}{h}\int_{-h/2}^{h/2} \cos^2 2\theta(z)\, dz = \int_0^1 \cos^2 2\theta(u)\, du$$

$$\xi_9 \equiv \frac{12}{h^3}\int_{-h/2}^{h/2} \cos 2\theta(z) z^2\, dz = 3\int_0^1 \cos 2\theta(u) u^2\, du$$

$$\xi_{10} \equiv \frac{12}{h^3}\int_{-h/2}^{h/2} \cos^2 2\theta(z) z^2\, dz = 3\int_0^1 \cos^2 2\theta(u) u^2\, du \tag{4}$$

where $\theta(u)$ is a distribution function of the fiber direction through the thickness, and (ξ_1,ξ_2) and (ξ_9,ξ_{10}) represent the in-plane and out-of-plane lamination parameters, respectively. The lamination parameters not only characterize the laminate configurations but also govern the stiffness characteristics of the laminate.

In Eqs. (2) and (3), the stiffness components are a linear function with respect to the lamination parameters. The buckling load for the laminated cylindrical shells is, for example, expressed by a much simpler function of the lamination parameters, as compared with the direct expression of the buckling load with respect to the layer angles. Thus, it leads to an efficient approach to use the lamination parameters as design variables in the stiffness optimization problem.

Fundamental Properties of Lamination Parameters

Reference 10 has shown the relation between four lamination parameters and the determining method of the laminate configurations corresponding to the lamination parameters. In this section, the fundamental properties of the lamination parameters are summarized.

The feasible regions of the in-plane or out-of-plane lamination parameters are, respectively, expressed as follows[2-4]:

$$-1 \le \xi_1 \le 1, \qquad \xi_1^2 \le \xi_2 \le 1$$
$$-1 \le \xi_9 \le 1, \qquad \xi_9^2 \le \xi_{10} \le 1 \tag{5}$$

where the relation of $\xi_2 \ge \xi_1^2$ in Eq. (5) can, for example, be derived from the condition of $\int_0^1 (\cos 2\theta - \xi_1)^2\, du \ge 0$.

Figure 1 shows the relation between the in-plane lamination parameters (ξ_1,ξ_2) and the n-layered laminate $[(\pm\theta_1)_{h_1}/(\pm\theta_2)_{h_2}/\cdots/(\pm\theta_n)_{h_n}]_s$. A point A_i on the parabola of $\xi_2 = \xi_1^2$ corresponds to an angle-ply laminate with the fiber angle $\pm\theta_i$. For example, the point B_1 corresponds to the 0-deg laminate, the point O to the $\pm$45-deg laminate, and the point C_1 to the 90-deg laminate. A point on the line of $\xi_2 = 1$ corresponds to a cross-ply laminate with the layer thicknesses of $h_0 = (1+\xi_1)/2$ and $h_{90} = (1-\xi_1)/2$. The point (ξ_1,ξ_2) = (0,½) corresponds to a quasi-isotropic laminate. A point P, with the lamination parameter (ξ_1,ξ_2), is expressed as a linear combination of the vectors a_i ($\cos 2\theta_i$, $\cos^2 2\theta_i$).

The feasible region of the out-of-plane lamination parameters (ξ_9,ξ_{10}) is restricted by the in-plane lamination parameters (ξ_1,ξ_2). The feasible region is obtained as follows. As shown in Fig. 2, the laminate configuration corresponding to the in-

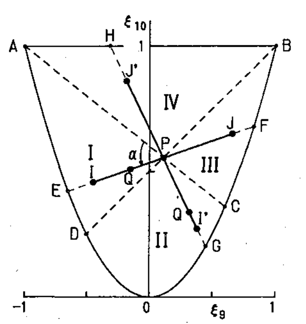


Fig 2 Relation between in-plane and out-of-plane lamination parameters.

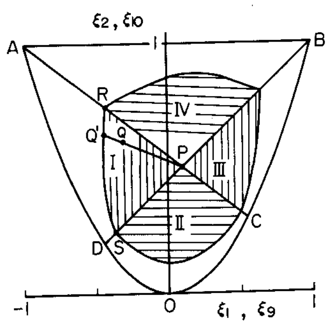


Fig 3 Feasible region of out-of-plane lamination parameters.

plane lamination parameters, $P(\xi_1,\xi_2)$, is expressed as a linear combination of two vectors on the boundary: $\boldsymbol{e}$ (point E) and $\boldsymbol{f}$ (point F) or $\boldsymbol{g}$ (point G) and $\boldsymbol{h}$ (point H). As the angle $\alpha = \angle APE$ is varied from 0 to π, the feasible region of the out-of-plane lamination parameters on the line EF and GH are represented by IJ and $I'J'$, respectively. The boundaries of the regions I–IV, shown in Fig. 3, are expressed as follows:

$$\xi_{10} = \frac{\xi_2 - \xi_E^2}{\xi_1 - \xi_E}\,\xi_9 + \frac{\xi_1\xi_E - \xi_2}{\xi_1 - \xi_E}\,\xi_E \tag{6a}$$

where Region I

$$\left(-1 \le \xi_E \le \frac{\xi_1 - \xi_2}{1 - \xi_1} \right)$$

$$\xi_9 = \xi_E + \frac{1}{\xi_1 - \xi_E}\,\frac{(\xi_1^2 - 2\xi_1\xi_E + \xi_E^2)^3}{(\xi_2 - 2\xi_1\xi_E + \xi_E^2)^2}$$

Region III

$$\left(-1 \le \xi_E \le \frac{\xi_1 - \xi_2}{1 - \xi_1} \right)$$

$$\xi_9 = \xi_E + (\xi_1 - \xi_E)\left[1 + \frac{\xi_2 - \xi_1^2}{\xi_2 - 2\xi_1\xi_E + \xi_E^2} + \left(\frac{\xi_2 - \xi_1^2}{\xi_2 - 2\xi_1\xi_E + \xi_E^2} \right)^2 \right]$$

Region II

$$\left(\frac{\xi_1 - \xi_2}{1 - \xi_1} \le \xi_E \le \frac{\xi_1 + \xi_2}{1 + \xi_1} \right)$$

$$\xi_9 = \xi_E + \frac{(\xi_2 - \xi_E^2)^2}{(1 - \xi_E^2)^2}\,(\xi_1 - \xi_E)$$

Region IV

$$\left(\frac{\xi_1 - \xi_2}{1 - \xi_1} \le \xi_E \le \frac{\xi_1 + \xi_2}{1 + \xi_1} \right)$$

$$\xi_9 = \xi_E + \frac{(1 - \xi_E^2)(\xi_1 - \xi_E)}{\xi_2 - \xi_E^2}\left[1 - \left(\frac{1 - \xi_2}{1 - \xi_E^2} \right)^3 \right] \tag{6b}$$

where ξ_E is a parameter relating ξ_9 in Eq. (6b) to ξ_{10} in Eq. (6a) and the point (ξ_E,ξ_E^2) corresponds to the point E for $0 \le \alpha \le \angle APD$ and to the point G for $\angle APD \le \alpha \le \pi$.

Next, we obtain the laminate configuration corresponding to the lamination parameters $(\xi_1,\xi_2,\xi_9,\xi_{10})$ in the feasible region. In the regions I and III of Fig. 2, the laminate configurations are expressed by the $(\pm\theta_E)/(\pm\theta_F)$ laminate, which corresponds to two points E and F obtained as the intersection of the line PQ and the parabola. In the regions II and IV, the laminate configurations are expressed as the $(\pm\theta_G)/0/90$ laminate, which corresponds to two points G and H obtained as the intersection of the line PQ and the parabola, and the intersection of the line PQ and the line AB. When the laminate configurations are represented by $[(\pm\theta_E)_{h_1}/(\pm\theta_F)_{h_2}/(\pm\theta_E)_{h_3}]_s$ laminates in regions I and III and $[(\pm\theta_G)_{h_1}/0_{h_2}/90_{h_3}/0_{h_4}/(\pm\theta_G)_{h_5}]_s$ laminates in regions II and IV, the layer angles and the layer thicknesses are given as follows[10]:

$$[(\pm\theta_E)_{h_1}/(\pm\theta_F)_{h_2}/(\pm\theta_E)_{h_3}]_s \tag{7a}$$

in regions I and III, where

$$\theta_i = ½ \cos^{-1}\xi_i \qquad (i = E,F)$$

$$\begin{Bmatrix} \xi_E \\ \xi_F \end{Bmatrix} = \frac{1}{2}\left[\frac{\xi_{10} - \xi_2}{\xi_9 - \xi_1} \mp \sqrt{\left(\frac{\xi_{10} - \xi_2}{\xi_9 - \xi_1} \right)^2 - 4\left(\frac{\xi_1\xi_{10} - \xi_2\xi_9}{\xi_9 - \xi_1} \right)} \right]$$

$$h_1 = \frac{\xi_F - \xi_1}{\xi_F - \xi_E} - h_3, \qquad h_2 = \frac{\xi_1 - \xi_E}{\xi_F - \xi_E}$$

$$h_3 = \frac{1}{6}\sqrt{12\left(\frac{\xi_9 - \xi_E}{\xi_1 - \xi_E} \right) - 3h_2^2} - \frac{h_2}{2} \tag{7b}$$

where ξ_E and ξ_F are given by the intersection of the line PQ and the parabola, as shown in Fig. 2. When the point P coincides with the point Q, ξ_E can be assigned to an arbitrary value satisfying $-1 \le \xi_E \le (\xi_1 - \xi_2)/(1 - \xi_1)$.

$$[(\pm\theta_G)_{h_1}/0_{h_2}/90_{h_3}/0_{h_4}/(\pm\theta_G)_{h_5}]_s \tag{8a}$$

in regions II and IV, where

$$\theta_G = ½ \cos^{-1}\xi_G$$

$$\xi_G = \xi_F \quad \text{for} \quad (\xi_{10} - \xi_2)/(\xi_9 - \xi_1) < 0$$

$$\xi_G = \xi_E \quad \text{for} \quad (\xi_{10} - \xi_2)/(\xi_9 - \xi_1) > 0$$

$$h_5 = \frac{1}{6}\sqrt{12\,\frac{\xi_9 - \xi_G}{\xi_1 - \xi_G} - 3\left(\frac{\xi_2 - \xi_G^2}{1 - \xi_G^2} \right)^2} - \frac{1}{2}\,\frac{\xi_2 - \xi_G^2}{1 - \xi_G^2}$$

$$h_1 = \frac{1 - \xi_2}{1 - \xi_G^2} - h_5$$

$$\xi_0 = \xi_1 + \frac{(1 - \xi_2)(\xi_9 - \xi_1)}{\xi_{10} - \xi_2}$$

$$a = \frac{1 - \xi_G^2}{\xi_2 - \xi_G^2}\,[(1 - h_1)^3 - h_5^3]$$

$$h_3 = \frac{1}{2}\,\frac{(1 - \xi_0)(\xi_2 - \xi_G^2)}{(1 - \xi_G^2)}, \qquad h_4 = \frac{1}{6}\sqrt{12a - 3h_3^2} - \frac{1}{2}h_3 - h_5$$

$$h_2 = \frac{1}{2}\,\frac{(1 + \xi_0)(\xi_2 - \xi_G^2)}{(1 - \xi_G^2)} - h_4 \tag{8b}$$

where ξ_E and ξ_F are given in Eq. (7b).

The laminate configurations shown in Eqs. (7) and (8) correspond to the laminate with the least number of layers, and the layer thicknesses are assumed to have the continuous value. When the number of plies is finite, we can obtain the laminate configurations with four different kinds of ply angles, as shown in Ref. 12.

Buckling Optimization of Orthotropic Laminated Cylindrical Shells

Fundamental Equation

The Donnell's governing equation for the buckling of orthotropic laminated cylindrical shells in Fig. 4 is

$$D_{11}w_{,xxxx} + 2(D_{12} + 2D_{66})w_{,xxyy} + D_{22}w_{,yyyy} + (1/R)U_{,xx} + \bar{N}_x w_{,xx} + \bar{N}_y w_{,yy} = 0 \tag{9}$$

$$a_{22}U_{,xxxx} + (2a_{12} + a_{66})U_{,xxyy} + a_{11}U_{,yyyy} - (1/R)w_{,xx} = 0$$

where $\bar{N}_x$ and $\bar{N}_y$, respectively, denote the compressive loads in the axial and circumferential directions; w and U, respectively, the deflection and the stress function; R, h, and L denote the radius, the thickness, and the length of the cylindrical shell, respectively; a comma denotes the differentiation with respect to the subscript. The out-of-plane stiffnesses D_{ij} are given in Eq. (3) and the in-plane compliances a_{ij} can be given by the inverse matrix of A_{ij}.

We assume the $S-2$ simply-supported boundary conditions ($N_x = v = w = M_x = 0$) at both edges, $x = 0$ and L. The following deflection and stress functions satisfy the governing equation and the boundary conditions:

$$w = \bar{w}\sin(\lambda/R)x\cos n\theta, \qquad U = \bar{U}\sin(\lambda/R)x\cos n\theta \tag{10}$$

where $\lambda = m\pi R/L$, and $m(1,2,\ldots)$ and $n(0,1,\ldots)$, respectively, denote the number of half-waves in the axial direction and the number of full waves in the circumferential direction.

Introducing Eq. (10) and the lamination parameters into Eq. (9), the buckling stress is expressed as follows:

$$\bar{\sigma} \equiv \frac{P}{h} = \frac{h^2}{12R^2}\frac{\Phi_1}{k_x\lambda^2 + k_y n^2} + \frac{\lambda^4}{k_x\lambda^2 + k_y n^2}\frac{1}{\Phi_2} \tag{11}$$

where k_x and k_y denote the loading ratios of $k_x = \bar{N}_x/P$ and $k_y = \bar{N}_y/P$, respectively, and Φ_1 and Φ_2 are expressed as follows:

$$\Phi_1 = (U_1 + \xi_9 U_2 + \xi_{10}U_3)\lambda^4 + 2(U_1 + 2U_3 - 3\xi_{10}U_3)\lambda^2 n^2 + (U_1 - \xi_9 U_2 + \xi_{10}U_3)n^4$$

$$\Phi_2 = \phi_{22}\lambda^4 + (2\phi_{12} + \phi_{66})\lambda^2 n^2 + \phi_{11}n^4 \tag{12}$$

where

$$\phi_{11} = ha_{11} = (U_1 - \xi_1 U_2 + \xi_2 U_3)/\gamma$$

$$\phi_{12} = ha_{12} = -(U_4 - \xi_2 U_3)/\gamma$$

$$\phi_{22} = ha_{22} = (U_1 + \xi_1 U_2 + \xi_2 U_3)/\gamma$$

$$\phi_{66} = ha_{66} = 1/(U_5 - \xi_2 U_3)$$

$$\gamma = (U_1 + U_4)(U_1 + 2\xi_2 U_3 - U_4) - \xi_1^2 U_2^2 \tag{13}$$

The buckling stress is given by the minimum of Eq. (11) with respect to the wave numbers m and n. The buckling stress is normalized as follows:

$$f_{\min} = \frac{1}{E_L}\min_{m,n}\bar{\sigma}(\xi_1,\xi_2,\xi_9,\xi_{10},m,n) \tag{14}$$

where E_L denotes the longitudinal Young's modulus. The normalized buckling stress $f_{\min}$ is a function of four lamination parameters.

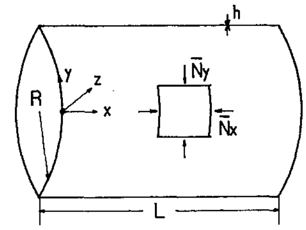


Fig 4 Configuration of laminated cylindrical shell.

Table 1 Elastic properties of graphite/epoxy composites

$E_L = 142$ GPa	$E_T = 10.8$ GPa
$G_{LT} = 5.49$ GPa	$\nu_L = 0.3$

Optimal Problem Formulation

We consider the buckling optimization problem of orthotropic laminated cylindrical shells under combined loadings. Two kinds of laminate constructions are considered. The first is the homogeneous laminate through the thickness, which satisfies the relation of $\xi_9 = \xi_1$ and $\xi_{10} = \xi_2$. Then, two lamination parameters ξ_1 and ξ_2 are used as design variables. The second is the heterogeneous laminate. Next, four lamination parameters are used as design variables.

The optimization problem is stated as follows:
Objective function

$$F = \max_{\xi_i} f_{\min} \tag{15a}$$

Constraint function

$$g_1 = \xi_1^2 - \xi_2 \le 0$$

$$-1 \le \xi_1 \le 1, \quad \xi_2 \le 1 \quad \text{for homogeneous laminates} \tag{15b}$$

$$g_1 = \xi_1^2 - \xi_2 \le 0, \qquad g_2 = PQ/PQ' - 1 \le 0$$

$$-1 \le \xi_1 \le 1, \quad \xi_2 \le 1 \quad \text{for heterogeneous laminates} \tag{15c}$$

where PQ and PQ' in Eq. (15c) are shown in Fig. 3.

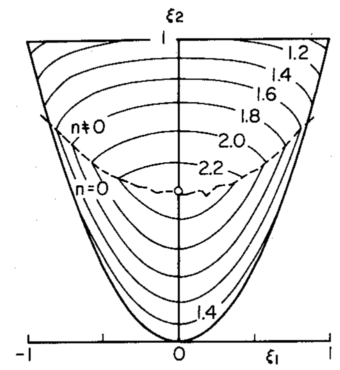
As an optimizer, the feasible direction method in automated design synthesis (ADS) program is used.[11]

Numerical Results and Discussions

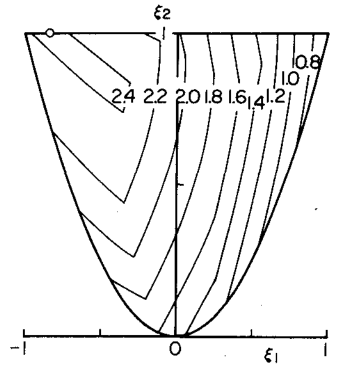
As numerical examples, graphite/epoxy composites are considered where the elastic properties are shown in Table 1. The geometry of cylindrical shells is $L/R = 2$ and $R/h = 100$. In the buckling calculation, the values of the wave numbers $m(1,2,\ldots,30)$ and $n(0,1,\ldots,20)$ are used for the axial compression ($k_x = 1$ and $k_y = 0$), and $m = 1$ and $n(0,1,\ldots,20)$ for the lateral loading ($k_x = 0$ and $k_y = 1$).

Buckling Characteristics of Laminated Cylindrical Shells

The buckling characteristics of the homogeneous cylindrical shells are governed by two lamination parameters ξ_1 and ξ_2. Figure 5 shows the contours of the normalized buckling stress $f_{\min}$ for the axial compression ($k_x = 1$ and $k_y = 0$) and for the lateral loading ($k_x = 0$ and $k_y = 1$). In the case of axial compression, the buckling modes are antisymmetric ($n \neq 0$) above the broken line while axis-symmetric ($n = 0$) below the broken line. In the case of lateral loading, the half-wave number m is equal to unity. The optimal point is indicated by a circle on the lamination parameter plane. The optimal laminate configurations are given by an isotropic laminate of (ξ_1,ξ_2) = (0,0.5) for the axial compression and a specific cross-ply laminate ($\xi_2 = 1$) for the lateral loading.



a) Axial compression $(k_x,k_y) = (1,0)$, $(f_{min} \times 10^3)$



b) Lateral loading $(k_x,k_y) = (0,1)$, $(f_{min} \times 10^4)$

Fig 5 Contours of the normalized buckling stresses on the lamination parameter plane.

Calculation of the Derivatives of Buckling Stresses

In the optimization process, the buckling stress derivative with respect to design variables is required. Figure 6 shows the contours of the buckling stress ($f_{min} \times 10^3$) for $(\xi_1,\xi_2) = (0.137, 0.202)$ in the case of axial compression. In the point O, two eigenvalues corresponding to the buckling mode $(m,n) = (1,5)$ and $(m,n) = (9,0)$ coincide. When the buckling stress derivative for the mode (1,5) is used, the gradient direction is OA and the optimization process may stop at the point O. In the present paper, the buckling stress gradient for the multiple eigenvalues is evaluated by a central difference approximation. This simple method gives a reliable gradient evaluation; in this example, the gradient direction at the point O becomes to OB direction.

Optimization Results

Homogeneous Laminates

The optimal results for the homogeneous laminated cylindrical shells are obtained under the loading conditions of $(k_x,k_y) = (1,0)$, $(1/\sqrt{2},1/\sqrt{2})$, and (0,1). Six different initial points are used for each loading condition. Those initial points converge to the almost same point. Table 2 shows the optimal laminate configurations corresponding to the optimal lamination parameters.

In the axial compression $(k_x,k_y) = (1,0)$, the optimal laminate configurations are an isotropic laminate with respect to both in-plane and out-of-plane stiffnesses, as obtained by Onoda.[9] When we use Eq. (7a), the value of ξ_E can be selected as an arbitrary value satisfying $-1 \le \xi_E \le 1$, since $(\xi_9,\xi_{10}) = (\xi_1,\xi_2)$. As examples of isotropic laminates, the laminate configurations corresponding to $\xi_E = -1, -½, ½$, and 1 are shown in Table 2. When the laminate configurations are expressed by the laminate with the finite number of plies as shown in Ref. 12, the examples of isotropic laminates consisting of $[(\pm\theta_1)/(\pm\theta_2)_2/(\pm\theta_3)/(\pm\theta_4)]_s$ are shown in Table 2. The bending-twisting coupling terms, D_{16} and D_{26}, in the laminates shown in Table 2a are not exactly zero. The

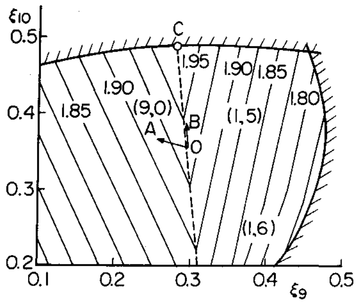


Fig 6 Evaluation of buckling stress derivatives.

Table 2 Optimal laminate configurations for homogeneous shells

a) $(\xi_1,\xi_2) = (0.000,0.500)$ for $(k_x,k_y) = (1,0)$

$[(\pm\theta_1)_{h_1}/(\pm\theta_2)_{h_2}/(\pm\theta_1)_{h_3}]_s$ laminates Eq. (7a)

θ_1, deg	θ_2, deg	h_1	h_2	h_3
90.0	30.0	0.122	0.667	0.211
60.0	0.0	0.264	0.333	0.403
30.0	90.0	0.264	0.333	0.403
0.0	60.0	0.122	0.667	0.211

$[(\pm\theta_1)/(\pm\theta_2)_2/(\pm\theta_3)/(\pm\theta_4)]_s$ laminates Ref. 12

θ_1, deg	θ_2, deg	θ_3, deg	θ_4, deg
63.5	19.5	82.2	44.7
26.5	70.5	7.8	45.3
23.1	69.7	47.9	11.1
66.9	20.3	42.1	78.9

b) $(\xi_1,\xi_2) = (-0.848,1.000)$ for $(k_x,k_y) = (1/\sqrt{2},1/\sqrt{2})$ and (0,1)

$[90_{0.385}/0_{0.076}/90_{0.539}]_s$

isotropic laminate configurations eliminating the coupling terms have been obtained in Ref. 12.

In the loading conditions of $(k_x,k_y) = (1/\sqrt{2},1/\sqrt{2})$ and (0,1), the optimal lamination parameters are $(\xi_1,\xi_2) = (-0.848, 1.000)$ and the corresponding laminate configuration is a specific cross-ply laminate, as shown in Table 2b. In this case, the laminate configurations cannot be expressed by the laminate with the finite number of plies, but the near-optimal laminate configurations can be obtained. For example, the buckling stresses f_{min} for the $[90_5/0/90_7]_s$ laminate are 3.104×10^{-4} for $(k_x,k_y) = (1/\sqrt{2},1/\sqrt{2})$ and 2.412×10^{-4} for $(k_x,k_y) = (0,1)$.

Heterogeneous Laminates

Table 3 shows the optimal results for the heterogeneous laminated cylindrical shells under the loading conditions of $(k_x,k_y) = (1,0)$ and (0,1). Six different initial points are used in the optimization. Maximum buckling stresses are almost the same, independent of the initial points, although the optimal lamination parameters depend a little on the initial points.

In the axial compression $(k_x,k_y) = (1,0)$, the optimal lamination parameters are $(\xi_1,\xi_2,\xi_9,\xi_{10}) = (0,0.5,0,0.5)$. The corresponding laminate configurations are the same as those in Table 2a.

In the loading conditions of $(k_x,k_y) = (1/\sqrt{2},1/\sqrt{2})$ and (0,1), the optimal lamination parameters $(\xi_1,\xi_2,\xi_9,\xi_{10})$ are $(-0.008, 0.709, -0.665, 0.902)$ and $(-0.070, 0.736, -0.727, 0.922)$, respectively. Those optimal lamination parameters are located on the boundary of the feasible region of the lamination parameters, as denoted by R in Fig. 3. The optimal laminate configurations using Eq. (7a) and near-optimal laminate configurations are shown in Table 4. Both in the cases of $(k_x,k_y) = (1/\sqrt{2},1/\sqrt{2})$ and (0,1), the maximum buckling stresses for the heterogeneous laminate are 27% higher than those for the homogeneous laminate. Thus, the effect of the stacking sequence on the buckling stress is remarkable.

Table 3 Optimal results of heterogeneous laminated cylindrical shells

a) $(k_x, k_y) = (1,0)$

	Lamination parameter				Buckling stress			
					Initial		Final	
Case	ξ_1	ξ_2	ξ_9	ξ_{10}	$F \times 10^3$	(m,n)	$F \times 10^3$	(m,n)
1	0.163	0.525	0.167	0.444	1.897	(1,5)	2.338	(2,7)
2	0.113	0.458	0.148	0.492	2.032	(5,9)	2.335	(2,7)
3	0.231	0.540	0.231	0.452	2.044	(10,0)	2.316	(2,7)
4	0.041	0.511	0.047	0.459	1.560	(8,0)	2.364	(2,7)
5	0.067	0.489	0.081	0.481	2.055	(4,8)	2.355	(2,7)
6	0.000	0.500	0.000	0.500	2.369	(5,9)	2.369	(3,8)

b) $(k_x, k_y) = (0,1)$

	Lamination parameter				Buckling stress			
					Initial		Final	
Case[a]	ξ_1	ξ_2	ξ_9	ξ_{10}	$F \times 10^4$	(m,n)	$F \times 10^4$	(m,n)
1	−0.070	0.736	−0.727	0.922	1.392	(1,6)	3.064	(1,5)
2	−0.188	0.785	−0.823	0.950	1.742	(1,6)	3.039	(1,5)
3	0.012	0.782	−0.673	0.931	1.531	(1,7)	3.047	(1,6)
4	−0.012	0.776	−0.696	0.931	1.333	(1,7)	3.060	(1,5)
5	−0.166	0.775	−0.808	0.948	2.188	(1,5)	3.047	(1,5)
6	−0.104	0.736	−0.754	0.924	1.924	(1,6)	3.059	(1,5)

[a]Initial points $(\xi_1, \xi_2, \xi_9, \xi_{10})$ used for each case
1) (0.1,0.2,0.3,0.4) 2) (0.1,0.7,0.2,0.6) 3) (0.3,0.4,0.3,0.4)
4) (0.6,0.5,0.4,0.3) 5) (−0.5,0.6,−0.4,0.6) 6) (0,0.5,0,0.5)

Table 4 Optimal laminate configurations for heterogeneous shells

a) $(\xi_1, \xi_2, \xi_9, \xi_{10}) = (-0.008, 0.709, -0.665, 0.902)$ for $(k_x, k_y) = (1/\sqrt{2}, 1/\sqrt{2})$

Laminate configurations	$f_{\min} \times 10^4$
Optimal	
$[90_{0.419}/(\pm 22.5)_{0.581}]_s$	3.944
Near-optimal	
$[90_4/(\pm 22.5)_6]_s$	3.939
$[90_5/(\pm 22.5)_5]_s$	3.927

(b) $(\xi_1, \xi_2, \xi_9, \xi_{10}) = (-0.070, 0.736, -0.727, 0.922)$ for $(k_x, k_y) = (0,1)$

Laminate configurations	$f_{\min} \times 10^4$
Optimal	
$[90_{0.458}/(\pm 22.1)_{0.542}]_s$	3.064
Near-optimal	
$[90_4/(\pm 22)_6]_s$	2.985
$[90_5/(\pm 22)_5]_s$	3.052

Conclusions

The present paper has shown an efficient approach to stiffness optimization of orthotropic laminated composites using the lamination parameters. The feasible regions of lamination parameters and the method of determining laminate configurations corresponding to the lamination parameters have been presented. When the lamination parameters are used as design variables, the stiffness optimization problem of laminated composites with respect to the layer orientation angles and the layer thickness ratios has been expressed by a simple expression with respect to design variables.

The present approach has been applied to the buckling optimization problems of the orthotropic laminated cylindrical shells under combined loadings. The optimal laminate configurations have been obtained efficiently and reliably.

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